Simplification of flapping forward flight

Ernesto Sánchez-Laulhé Cazorla, Ramón Fernández Feria

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Original simplification

- Characteristic magnitudes

$$U_c = \frac{\pi \rho S}{m\omega}; \quad L_c = \frac{c}{2}; \quad t_c = \frac{1}{\omega}$$

- Non-dimensional terms

$$\mathcal{M} = \omega \sqrt{\frac{m}{\pi \rho S g}}; \quad \chi = \frac{m g l_w}{\omega^2 I_y}; \quad k = \frac{\omega c}{2U_c}\frac{1}{U}$$

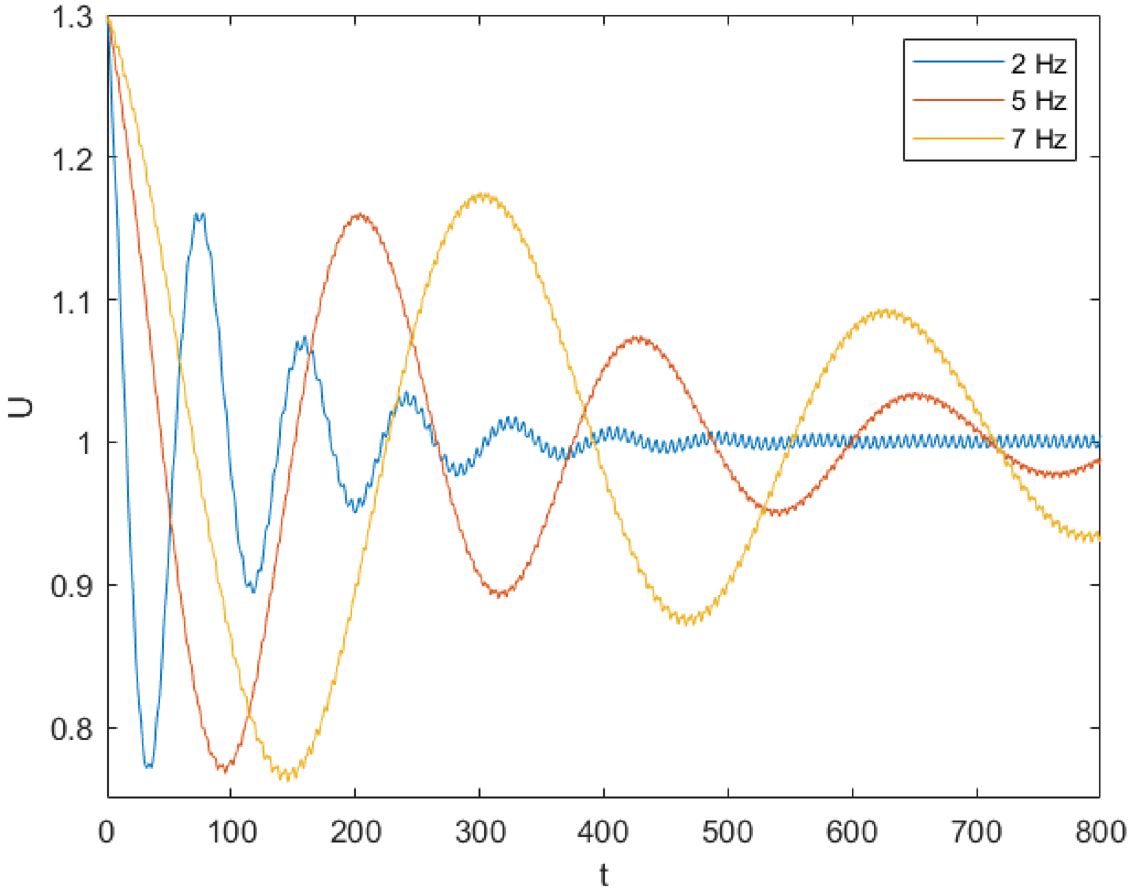
- Small parameter scale

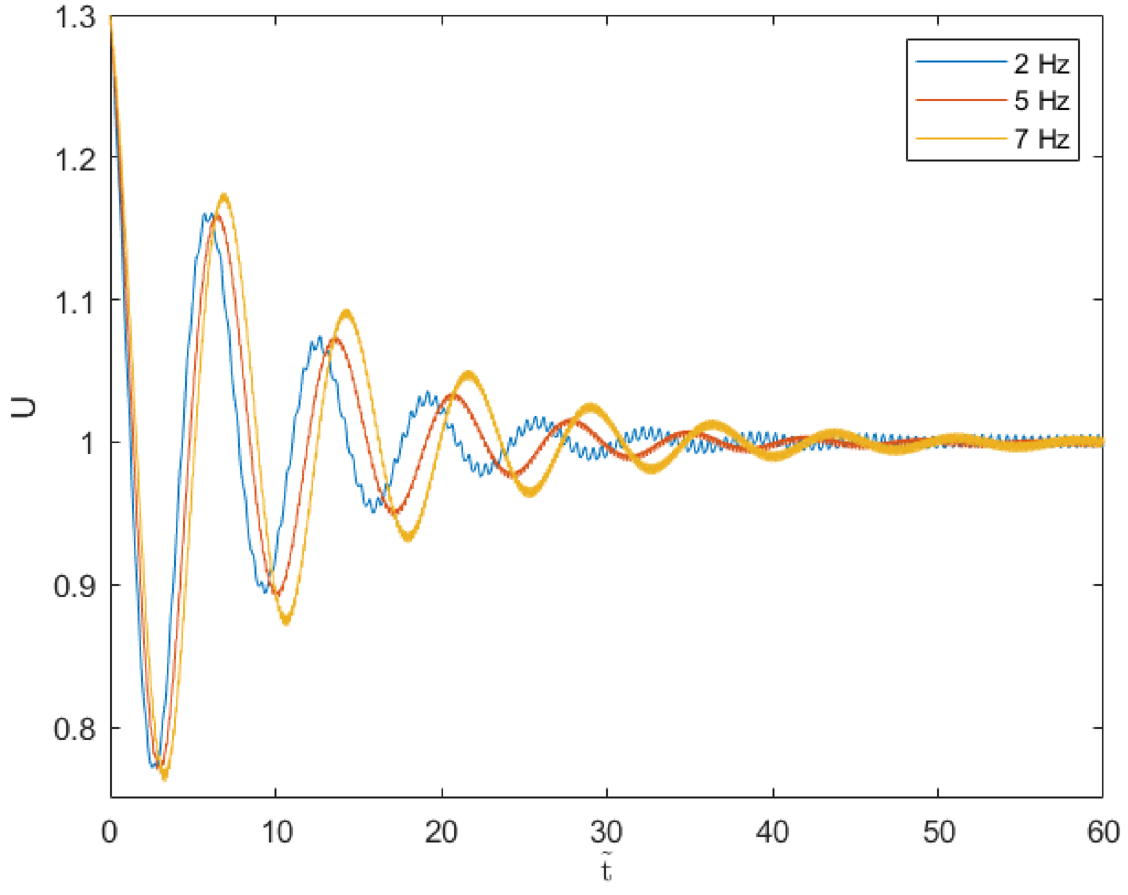
$$h_0 \to \epsilon; \quad \alpha, \gamma, \theta, \delta_t \sim \epsilon; \quad C_D \sim \epsilon$$

Damping

- Drag coefficient of order of the small parameter
- Problems
  - Small frequency range of validity
  - Thrust coefficient of smaller order
- New solution: New time scale slower for the damping
  - $C_D \sim \epsilon^2 \sim C_T$
  - Validity in a wider range
  - Influence of more terms in the damping of the system

Influence of the frequency





New definition of the equations

$$\mathcal{M}k_0\dot{U} = U^2\left(C_T^* - \left(C_{D0w}^* + C_{Di}^* + Li^* + \Lambda(C_{Dit}^* + C_{D0t}^*)\right)\right) + \delta_t \sin(\gamma)$$

$$\mathcal{M}k_0U\dot{\gamma} = U^2(C_L^* + \Lambda C_{Lt}^*) - \delta_t \cos(\gamma)$$

$$(\mathcal{M}k_0)^2\ddot{\theta} = \chi\mathcal{M}^2U^2\{\bar{l}_w C_L^* \cos(\alpha) - (C_T^* - C_{Dw}^*)\sin(\alpha) + \bar{l}_t\Lambda[C_{Lt}^* \cos(\alpha) + C_{Dt}\sin(\alpha)] - \bar{h}_w[C_L^* \sin(\alpha) + (C_T^* - C_{Dw}^*)\cos(\alpha)]\}$$

New definition of the equations

- Characteristic magnitudes

$$U_c = \sqrt{\frac{mg}{\pi\rho S\epsilon}}; \quad L_c = \frac{c}{2}; \quad t_c = \frac{1}{\omega}$$

- Non-dimensional terms

$$\mathcal{M} = \frac{2m}{\pi\rho Sc}; \quad \chi = \frac{\pi\rho S\left(\frac{c}{2}\right)^3}{2I_y}; \quad k_0 = \frac{\omega c}{2U_c}$$

- Small parameter scale

$$k_0 h_0, \delta_t \rightarrow \epsilon; \quad \alpha, \gamma, \theta \sim \epsilon$$

Parameters order

- Airspeed of order 1
- Non-dimensional terms of order 1
- Drag terms of order ϵ^2

$\tilde{f}$	$\mathcal{M}k_0$	$\mathcal{M}$	$\mathcal{M}^2\chi$	Λ	l_w	$l_t\Lambda$	h_w	Li^*	AR
$2\ Hz$	0.64	1.72	0.97	0.21	0.34	-0.68	0.38	0.0016	5.14
$5\ Hz$	1.59								
$7\ Hz$	2.23								

Time scales

- Flapping time scale t
  - Oscillations
  - Short period (neglected if stable)
- Phugoid oscillations

$$\tau_1 = \frac{\epsilon}{\mathcal{M} k_0}$$

- Phugoid damping

$$\tau_2 = \frac{\epsilon^2}{\mathcal{M} k_0}$$

Solutions

- State variables

$$\alpha = \epsilon\left[A_0(\tau_1,\tau_2)e^{it} + A_1(\tau_1,\tau_2) + \epsilon\left(A_2(\tau_1,\tau_2)e^{it} + A_3(\tau_1,\tau_2)e^{2it} + A_4(\tau_1,\tau_2)\right) + \cdots\right]$$

$$U = U_0(\tau_1,\tau_2) + \epsilon U_1(\tau_1,\tau_2) + \epsilon^2\left(U_2(\tau_1,\tau_2) + V_1(\tau_1,\tau_2)e^{it} + V_2(\tau_1,\tau_2)e^{2it}\right) + \cdots$$

$$\theta = \epsilon\left[T_0(\tau_1,\tau_2)e^{it} + T_1(\tau_1,\tau_2) + \epsilon\left(T_2(\tau_1,\tau_2)e^{it} + T_3(\tau_1,\tau_2)e^{2it} + T_4(\tau_1,\tau_2)\right) + \cdots\right]$$

- Aerodynamic coefficients

$$C_L^* = a_1\alpha + \frac{a_2}{U}h_0e^{it}; \quad C_{Lt} = a_3(\alpha + \delta_t) + \frac{a_4}{U}\frac{\partial\theta}{\partial t} + \frac{a_5}{U}\frac{\partial\alpha}{\partial t}$$

$$C_T^* = b_1\alpha^2 + \frac{b_2}{U}\alpha h_0e^{it} + \frac{b_3}{U^2}h_0 + \frac{C_{Th}}{U^2}$$

Solutions

- First system: mean terms for airspeed and angle of attack

$$U_0 = \sqrt{\frac{1}{a_1 A_1 + a_3 \Lambda (A_1 + \delta_t^*)}}$$

$$A_1 = -\frac{\bar{l}_t \Lambda a_3 \delta_t^*}{\bar{l}_w a_1 + \bar{l}_t \Lambda a_3 \delta_t^*}$$

- Second system: Amplitude of order ϵ oscillations

$$i(\mathcal{M} k_0 - \Lambda a_4) T_0 + \left(-i\mathcal{M} k_0 - i\Lambda a_5 - U_0(a_1 + \Lambda a_3)\right) A_0 = a_2$$

$$-\left(\frac{(\mathcal{M} k_0)^2}{\mathcal{M}^2 \chi} + iU_0 \bar{l}_t \Lambda a_4\right) T_0 - \left(U_0^2(\bar{l}_w a_1 + \bar{l}_t \Lambda a_3) + iU_0 \bar{l}_t \Lambda a_5\right) A_0 = \bar{l}_w U_0 a_2$$

Solutions

- Short period equations

$$\mathcal{M}k_0U_0\left(\frac{\partial\theta_0}{\partial t}-\frac{\partial\alpha_0}{\partial t}\right)=U_0^2\big(a_1\alpha_0+\Lambda(a_3\alpha_0)\big)+U_0\Lambda\left(a_4\frac{\partial\theta_0}{\partial t}+a_5\frac{\partial\alpha_0}{\partial t}\right)$$

$$(\mathcal{M}k_0)^2\frac{\partial^2\theta_0}{\partial t^2}=\mathcal{M}^2\chi U_0^2\left(\bar{l}_w a_1\alpha_0+\bar{l}_t\Lambda(a_3\alpha_0)\right)+\mathcal{M}^2\chi U_0\bar{l}_t\Lambda\left(a_4\frac{\partial\theta_0}{\partial t}+a_5\frac{\partial\alpha_0}{\partial t}\right)$$

- Second system: constant terms of U_1, T_1, A_4

$$T_{1c}=A_1+U_0^2\left((b_1+\Lambda b_4)(A_1^2+2A_0\bar{A}_0)+\Lambda b_5A_1\delta_t^*-C_D^*\right)+U_0\big(b_2\bar{A}_0+\bar{b}_2A_0\big)+C_{Th}^*$$

$$U_{1c}=-A_{4c}\frac{U_0}{2}\frac{a_1+\Lambda a_3}{a_1A_1+a_3\Lambda(A_1+\delta_t^*)}$$

$$A_{4c}=\frac{\bar{h}_w\left((b_1+a_1)(A_1^2+2A_0\bar{A}_0)-C_{Dw}^*\right)+\frac{1}{U_0}\left((a_2+b_2)\bar{A}_0+\big(\bar{a}_2+\bar{b}_2\big)A_0\right)+\frac{C_{Th}^*}{U_0^2}}{\left(\bar{l}_w a_1+\bar{l}_t\Lambda a_3\right)}$$

Solutions

- Differential equations: phugoid frequency and phase

$$T_1(\tau_1,\tau_2) = e^{i\phi_T} e^{i\omega_1 \tau_1} T_1(\tau_2); \quad U_1(\tau_1,\tau_2) = e^{i\phi_T} e^{i\omega_1 \tau_1} U_1(\tau_2)$$

$$A_4(\tau_1,\tau_2) = -\frac{\bar{l}_t \Lambda a_4 e^{i\phi_T} T_1(\tau_2) i\omega_1 e^{i\omega_1 \tau_1}}{U_0\left(\bar{l}_w a_1 + \bar{l}_t \Lambda a_3\right)}$$

$$T_1(\tau_2) i\omega_1 e^{i\phi_T} \left(1 - \Lambda a_4 \left(1 - \bar{l}_t \frac{a_1 + \Lambda a_3}{\bar{l}_w a_1 + \bar{l}_t \Lambda a_3}\right)\right) = 2U_1(\tau_2) e^{i\phi_U}\left(a_1 A_1 + a_3 \Lambda (A_1 + \delta_t^*)\right)$$

$$U_1(\tau_2) i\omega_1 e^{i\phi_U} = -T_1(\tau_2) e^{i\phi_T}; \quad U_1(\tau_2) e^{i\phi_U} = i\frac{T_1(\tau_2) e^{i\phi_T}}{\omega_1}$$

$$\omega_1 = \sqrt{2}\sqrt{\frac{a_1 A_1 + a_3 \Lambda (A_1 + \delta_t^*)}{1 - \Lambda a_4 \left(1 - \bar{l}_t \frac{a_1 + \Lambda a_3}{\bar{l}_w a_1 + \bar{l}_t \Lambda a_3}\right)}}; \quad \tan\phi_T = \omega_1 \frac{-U_i}{\theta_i}$$

Conclusions

- Time scale separation
  - Function of the frequency
  - Oscillations vary with frequency
  - Influence of the frequency in the pitch angle
  - Terms with the frequency disappear for phugoid characterisation
- Characteristic magnitudes
  - Time with flapping – Oscillations
  - Airspeed – Order 1 (function of the deflection of the tail)

Work to do

- Numerical validation
- Correction of terms
- Characterisation of the variables as a function of the parameters
- Aerodynamic coefficients redefinition
- Finish writing