

# Simplified Model for Forward-Flight Transitions of a Bio-Inspired Unmanned Aerial Vehicle

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**R o b o t i c s**

**V i s i o n**

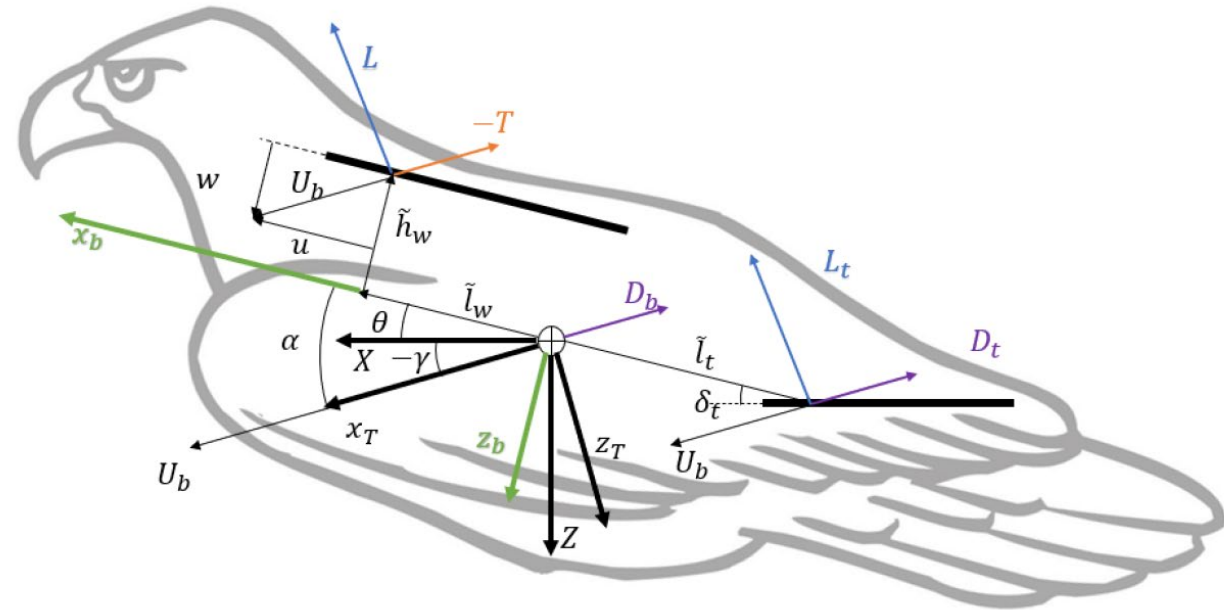
**C o n t r o l**

U n i v e r s i d a d  
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# Characteristics

- Longitudinal flight model
- Forces from linear potential theory
- Small angle of attack and amplitude
- Rigid body equations
- Two control variables: tail + flapping
- Formulation in the trajectory frame (Newton-Euler equations)



# Simplification: Non-dimensional parameters

- Velocity – Deflection of the tail
- Length – Half chord
- Time – Flapping frequency
- Reduced frequency – characteristic velocity
- Non-dimensional mass and inertia
- Modified force coefficients – order of magnitude of  $\alpha, h_0, \theta$

$$U_c = \sqrt{\frac{mg}{\pi\rho S\delta_t'}}, \quad L_c = \frac{c}{2}, \quad t_c = \frac{1}{\omega'}$$

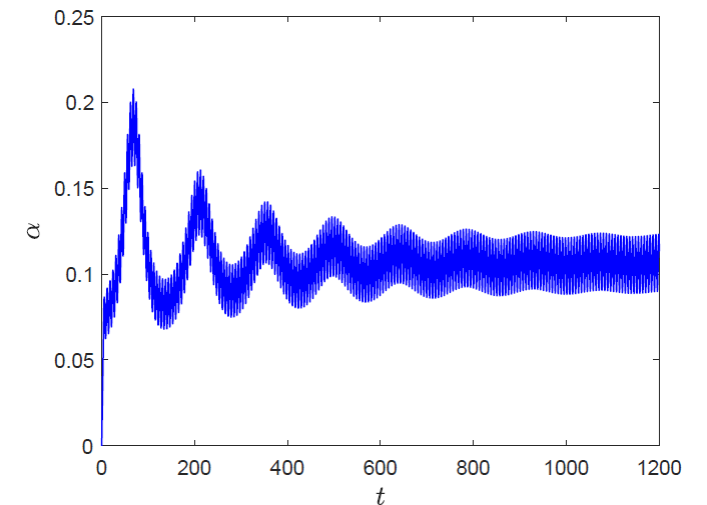
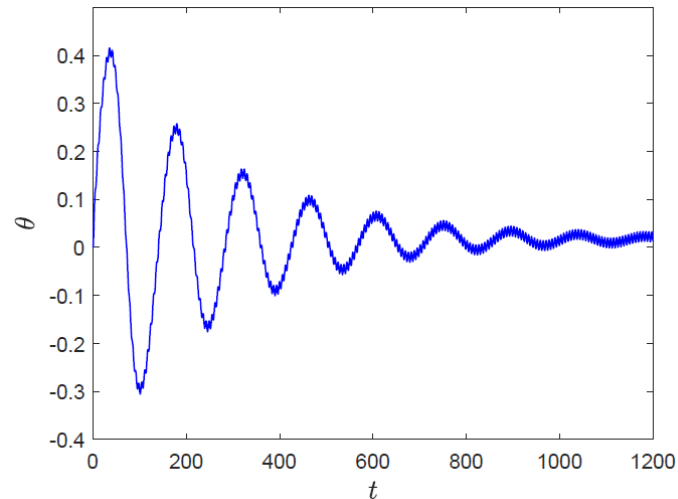
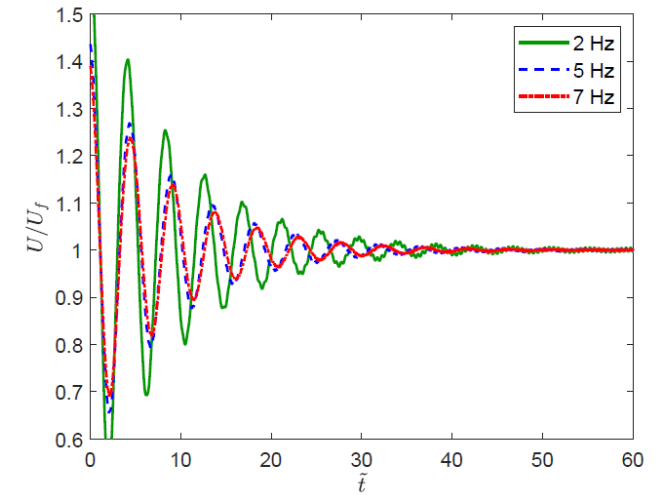
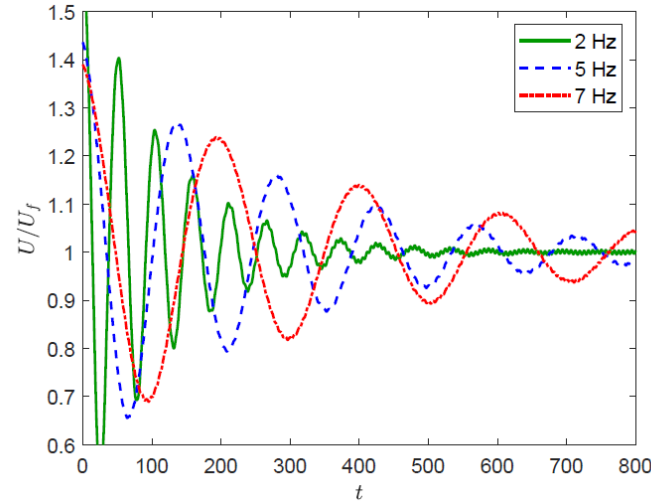
$$\mathcal{M} = \frac{2m}{\pi\rho S c'}, \quad \chi = \frac{\pi\rho S (\frac{c}{2})^3}{2I_y}, \quad k_0 = \frac{\omega c}{2U_c'}$$

$$C_L^* = \frac{L}{\pi\rho U_b'^2 c'}, \quad C_T^* = \frac{T}{\pi\rho U_b'^2 c'}, \quad C_D^* = \frac{D}{\pi\rho U_b'^2 c'}$$

# Numerical simulations

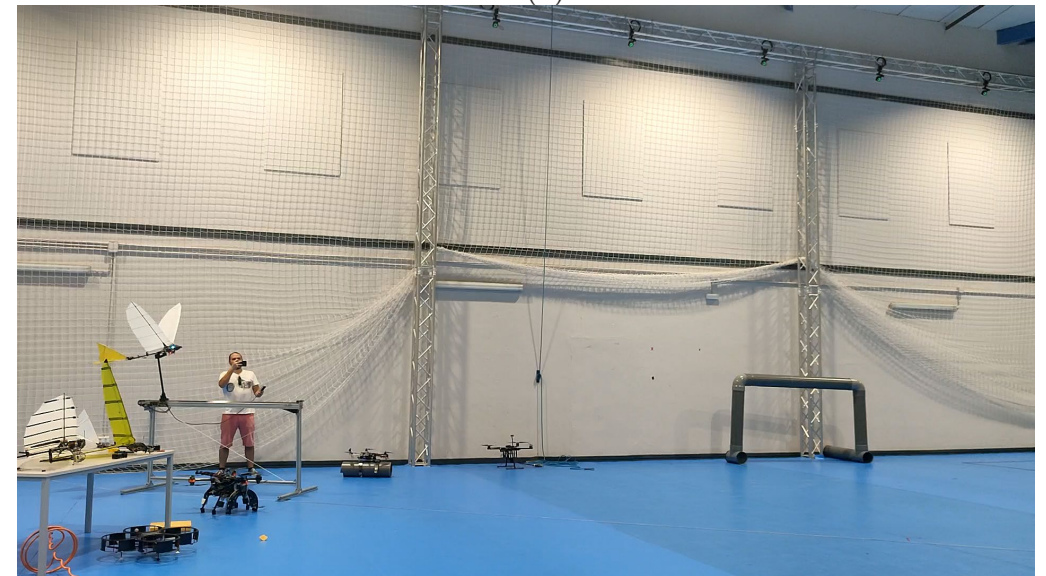
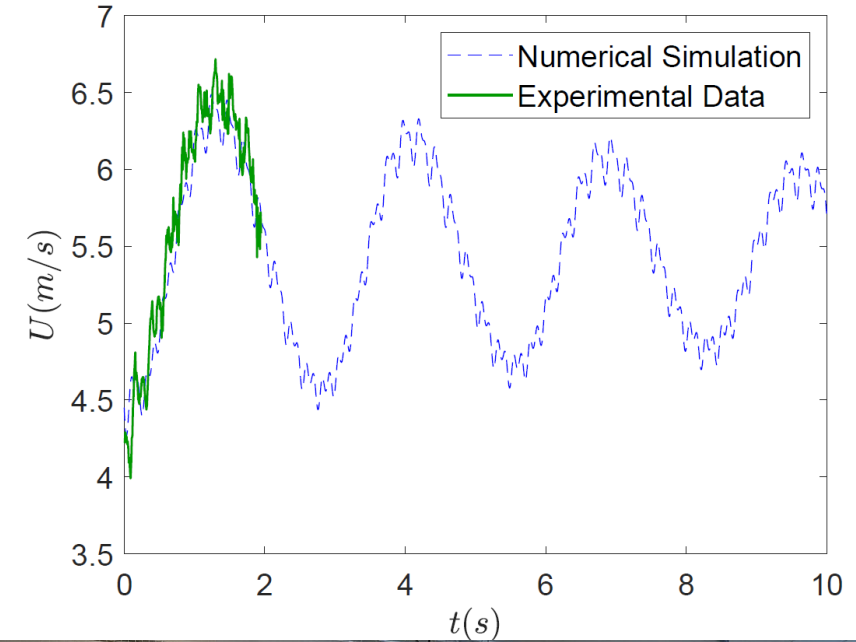
Simulations computationally expensive:

- Non-linear equations
- Dynamics-aerodynamics interaction
- Different timescales



# Experimental Validation

- Optitrack flights
- Rigid wings (thicker ribs)
- Open loop control (fixed for small angle of attack)
- Data obtained directly from motive
- Short flight time



# Scales analysis

- Small parameter:  $h_0 \equiv \epsilon$
- Small flight angles:  $\gamma \sim \theta \sim \alpha \sim \delta_t \sim \epsilon$
- Lift coefficients:  $C_L^* \sim C_{Lt}^* \sim \epsilon$
- Thrust coefficient:  $C_T^* \sim \epsilon^2$
- Drag coefficient:  $C_D^* \sim C_{Dt}^* \sim Li^* \sim \epsilon^2$
- Mass parameters:  $\mathcal{M}^2 \chi \sim \mathcal{M}k_0 \sim 1$
- First timescale (flapping oscillations):  
 $t$
- Second timescale (transient oscillations):  
 $\tau_{c1} = \mathcal{M}k_0/\epsilon$
- Third timescale (transient damping)  
 $\tau_{c2} = \mathcal{M}k_0/\epsilon^2$

# Formulation

## Perturbative expansion

$$U = U_0(t, \tau_1, \tau_2) + \epsilon U_1(t, \tau_1, \tau_2) + \epsilon^2 U_2(t, \tau_1, \tau_2) + \dots$$

$$\theta = \epsilon(\theta_1(t, \tau_1, \tau_2) + \epsilon\theta_2(t, \tau_1, \tau_2) + \dots)$$

$$\alpha = \epsilon(\alpha_1(t, \tau_1, \tau_2) + \epsilon\alpha_2(t, \tau_1, \tau_2) + \dots)$$

$$\gamma = \epsilon[\theta_1(t, \tau_1, \tau_2) - \alpha_1(t, \tau_1, \tau_2)] + \epsilon^2[\theta_2(t, \tau_1, \tau_2) - \alpha_2(t, \tau_1, \tau_2)] + \dots$$

## Frequency expansion

$$U_0 = V_0(\tau_1, \tau_2) + V_1(\tau_1, \tau_2)e^{it} + \dots + V_n(\tau_1, \tau_2)e^{int}$$

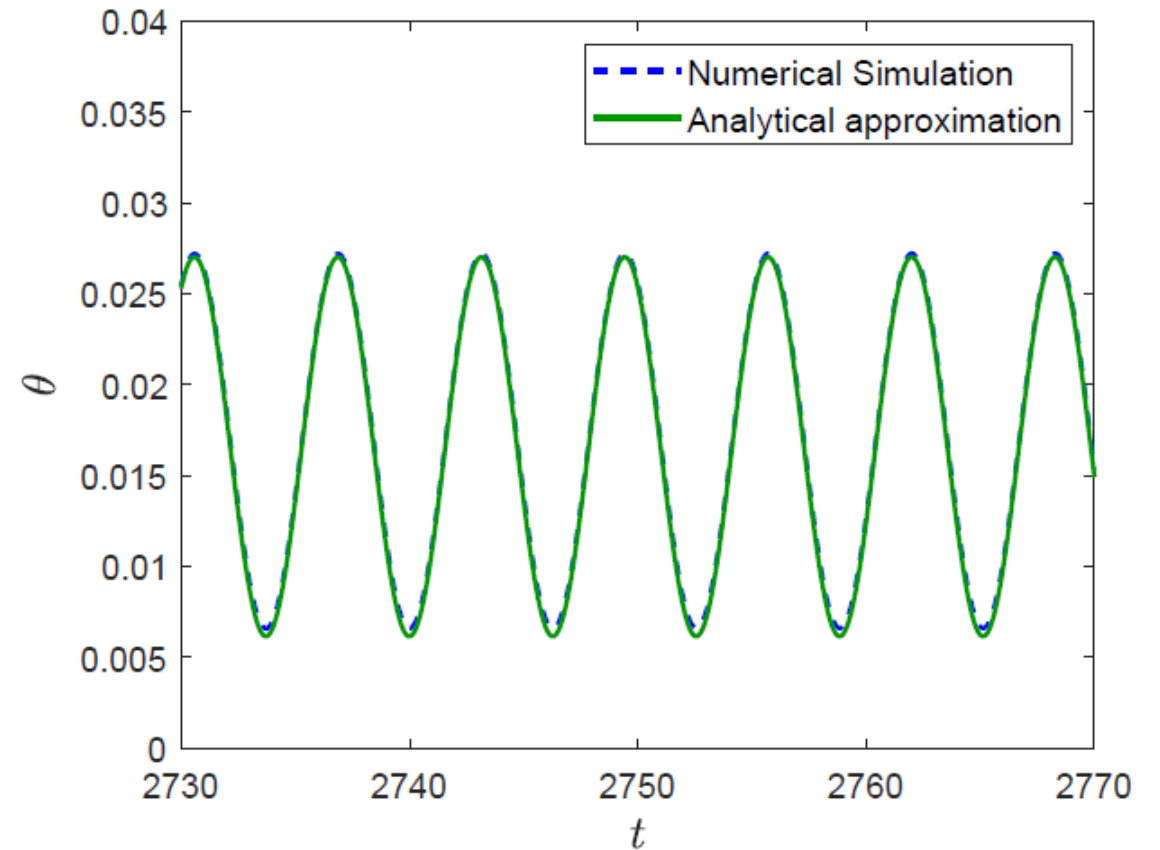
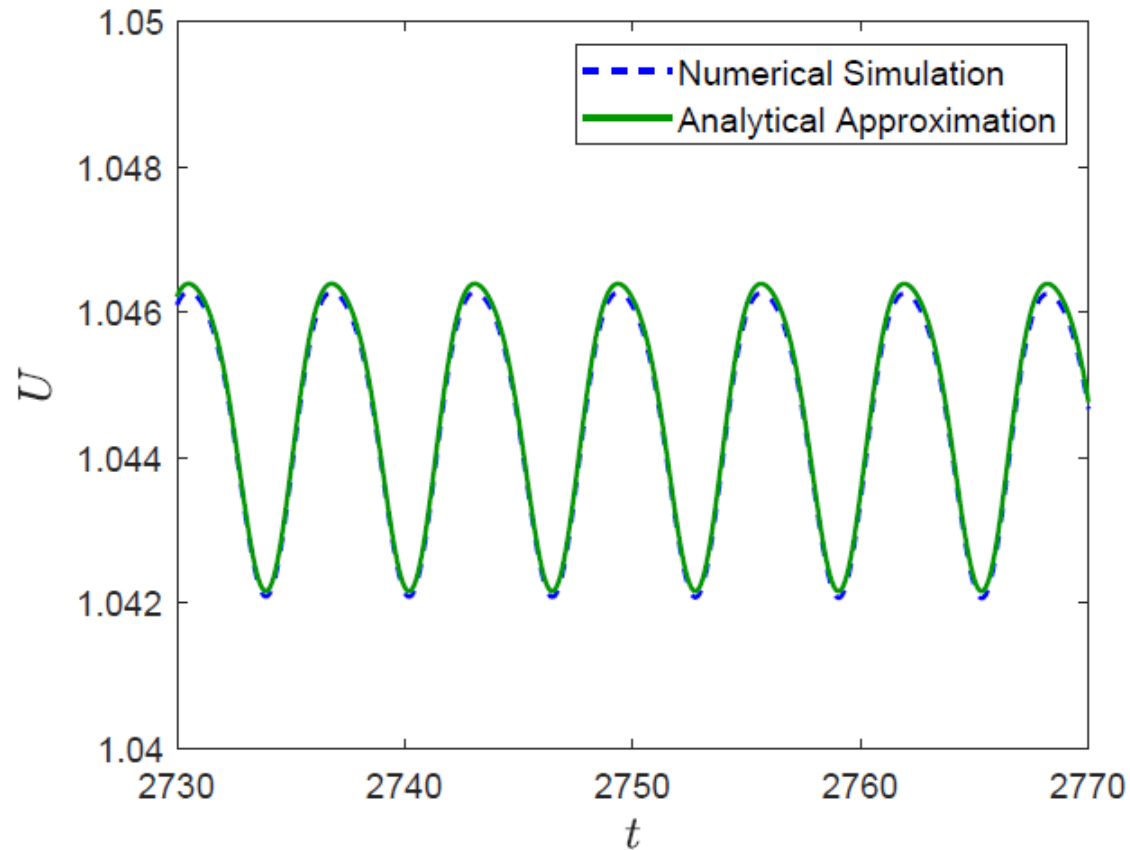
$$U_1 = V_{n+1}(\tau_1, \tau_2) + V_{n+2}(\tau_1, \tau_2)e^{it} + \dots + V_{n+k+1}(\tau_1, \tau_2)e^{ikt}$$

# Perturbation solution

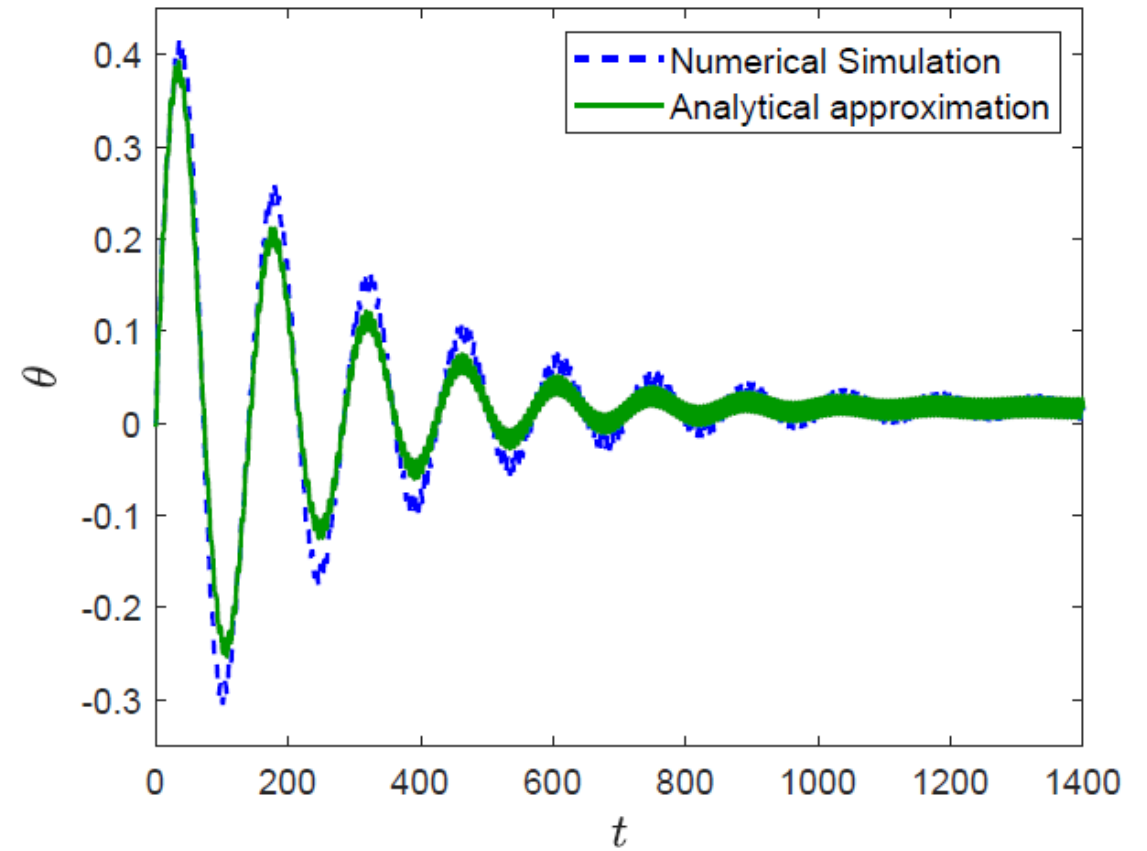
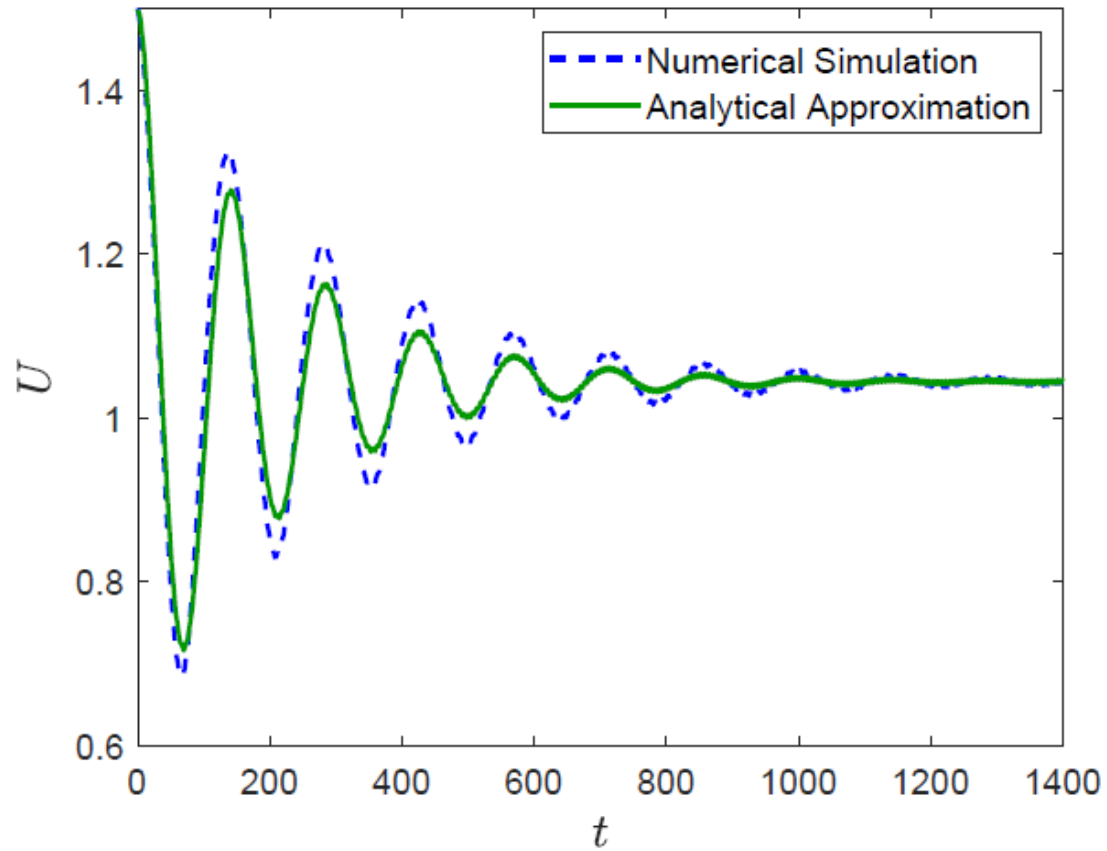
- First non-linear algebraical system
- Linear equations or system of equations
- Large-time asymptotic values: Terms up to order  $\epsilon^2$  for  $\theta$ , and order  $\epsilon^3$  for  $U$  and  $\alpha$
- Transient terms: Terms up to order  $\epsilon$  for  $U$  and  $\theta$ , and order  $\epsilon^2$  for  $\alpha$
- Harmonic oscillations: Terms up to order  $\epsilon^2$  for the three variables



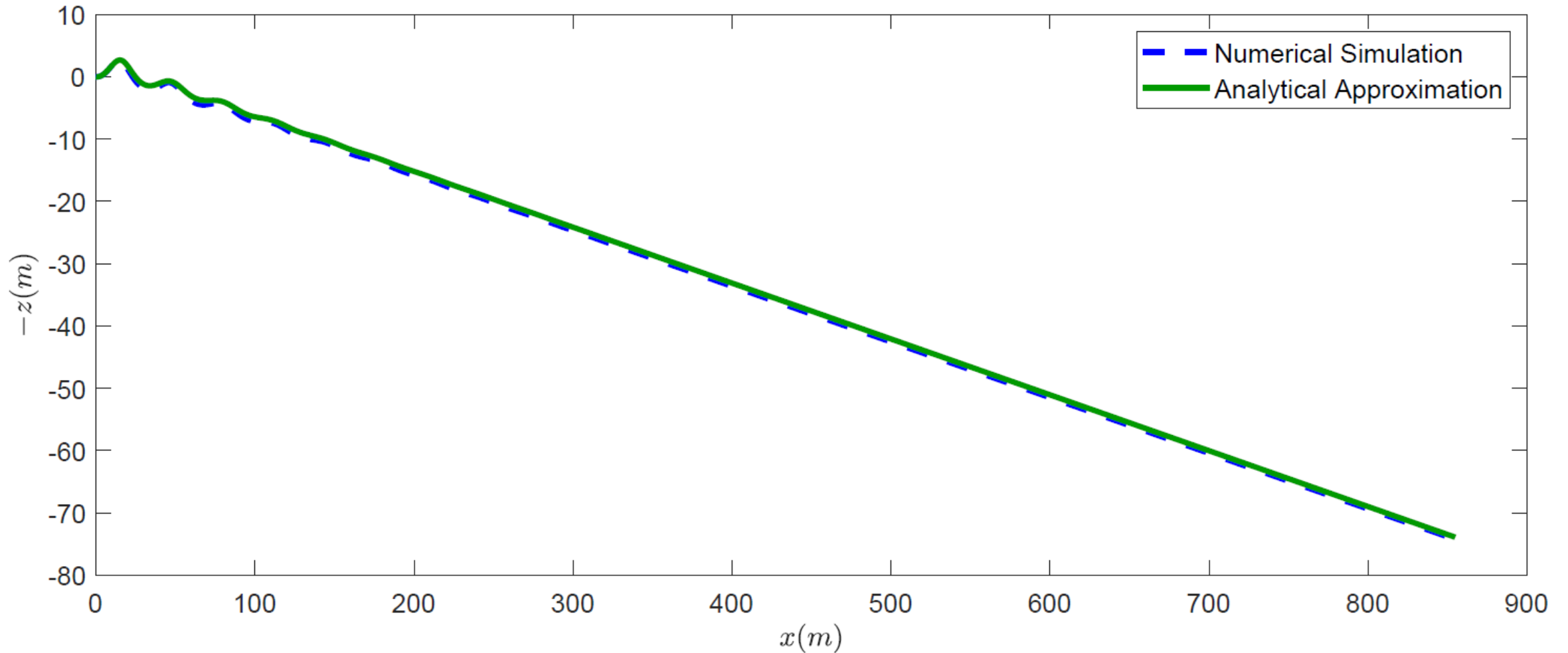
# Results: permanent state



# Results: transient phase



# Results: trajectory



# Conclusion

- Simplified model: reduced to linear equations
- First model with results from aerodynamic potential theory
- Analytical solution of the flapping permanent state
- Direct insight into the influence of control and design variables
- Model defined for cruise flight: no aggressive maneuvers
- Simplification can be generalized for pitching and flexible wings